Sheet 2

First Order Ordinary Differential Equations

1) Solve the differential equation $\frac{dy}{dx} = x^2y + 2x,\ \ y(0) = 0$ using **Huen's method** to find y(2) and y(5).

2) Solve the initial value problem $\frac{dy}{dx} = 0.5\,(1 + x)\,y^2,\ \ y(0) = 1$, for x = 1 and x=6 using **Huen's method** formula.

3) Solve the following problem numerically from t = 0 to 2:

$$\frac{dy}{dt} = -y + t \qquad y(0) = 1$$

Use the **Huen's** method with a step size of 1.

4) Find y(0.1) using **Runge-Kutta method** when $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1. Take step size = 0.05.

5) Solve the initial value problem $\frac{dy}{dx} = x\,y + y^2$ given that y(0) = 1 for x = 0.1and 0.3 using **Runge-Kutta method**.

6) Solve the following initial-value problem over the interval from x = 0 to 1:

$$\frac{dy}{dx} = (1 + x)\sqrt{y} \qquad y(0) = 1$$

Use the **Runge-Kutta method** with a step size of 0.5.

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1 - السؤال الاول والرابع محلولين حل نموذجى
2 - السؤال الثانى والخامس سيتم شرحهم فى السكشن
3 - السؤال الثالث والسادس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة

$1 - Given$

$f(x,y) = \ x^2y \ + \ 2x$

$x_0 = 0$

$y_0 = 0$

$x_f = 2$

$Solution$

y_1

$h_1 = x_1 - x_0 = 2 - 0 = 2$

$y_1^e = y_0 + hf\ (x_0, y_0) = 0 + 2 * f(0,0) = 0$

$$y_1 = y_0 + h\frac{f(x_0,y_0) + f(x_1,y_1^e)}{2} = 0 + 2 * \frac{f(0,0) + f(2,0)}{2} = 4$$

y_2

$h_2 = x_2 - x_1 = 5 - 2 = 3$

$y_2^e = y_1 + hf\ (x_1, y_1) = 4 + 3 * f(2,4) = 64$

$$y_2 = y_1 + h\frac{f(x_1,y_1) + f(x_2,y_2^e)}{2} = 4 + 3 * \frac{f(2,4) + f(5,64)}{2} = 2449$$

x	y
0	0
2	4
5	2449

$4 - Given$
$f(x,y) = x^2 + y^2$
$x_0 = 0$
$y_0 = 1$
$x_f = 0.1$
$h = 0.05$

$Solution$

y_1
$x_1 = x_0 + h = 0 + 0.05 = 0.05$
$k_1 = f(x_0, y_0) = f(0,1) = 1$
$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hk_1}{2}\right) = f(0.025,1.025) = 1.051$
$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hk_2}{2}\right) = f(0.025,1.026) = 1.054$
$k_4 = f(x_0 + h, y_0 + hk_3) = f(0.05,1.053) = 1.111$
$y_1 = y_0 + h\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$
$= 1 + 0.05\frac{(1) + 2(1.051) + 2(1.054) + (1.111)}{6} = 1.053$

y_2
$x_2 = x_1 + h = 0.05 + 0.05 = 0.1$
$k_1 = f(x_1, y_1) = f(0.05,1.053) = 1.111$
$k_2 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{hk_1}{2}\right) = f(0.075,1.080) = 1.173$
$k_3 = f\left(x_1 + \frac{h}{2}, y_1 + \frac{hk_2}{2}\right) = f(0.075,1.082) = 1.176$
$k_4 = f(x_1 + h, y_1 + hk_3) = f(0.1,1.111) = 1.245$
$y_2 = y_1 + h\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$
$= 1.053 + 0.05\frac{(1.111) + 2(1.173) + 2(1.176) + (1.245)}{6} = 1.111$

x	y
0	1
0.050	1.053
0.100	1.111